

# The Impact of Rapid Wind Variability upon Air-Sea Thermal Coupling

Philip Sura and Matthew Newman

*CIRES Climate Diagnostics Center, University of Colorado  
and  
NOAA Earth System Research Laboratory  
Boulder, Colorado*

September 30, 2006

Submitted to *Journal of Climate*

*Corresponding author address:*

Philip Sura

NOAA-ESRL and CIRES Climate Diagnostics Center, R/PSD1

325 Broadway, Boulder, CO 80305-3328

Phone: (303) 497-4426, Fax: (303) 497-6449

E-Mail: Philip.Sura@noaa.gov

# Abstract

The basic effect of extratropical atmosphere-ocean thermal coupling is to enhance the variance of both anomalous sea surface temperatures (SST) and air temperatures (AIRT) due to a decreased energy flux between the atmosphere and ocean, called reduced thermal damping. In this paper it is shown that rapidly-varying winds have an important effect upon the turbulent surface heat fluxes that drive this coupling during wintertime, acting to effectively weaken the coupling and thus partially counteracting the reduced thermal damping.

The nonlinear relationship between the rapidly-varying wind speed anomalies and SST and AIRT anomalies results in a rapidly varying component of the surface heat fluxes. It is shown, however, that the clear separation between the dynamical timescales of the ocean and atmosphere allows a simple approximation of this rapidly varying flux by a stochastic process. In many previous studies such a stochastic term is approximated by Gaussian white-noise with fixed variance (that is, additive noise). However, as it is shown from first principles, this stochastic heat flux term depends upon not only the fast winds but also upon the more slowly-evolving thermal anomalies. Such state-dependent (multiplicative) noise can alter the dynamics of atmosphere-ocean coupling because it induces an additional heat flux term (the noise-induced drift). The noise-induced drift acts to effectively weaken coupling but also to effectively weaken dissipation.

A key consequence of the outlined theory is that air-sea coupling includes both deterministic and stochastic components. The hypothesis is tested by examining daily observations at several Ocean Weather Stations (OWSs). Two important results are found. First, mul-

tiplicative noise at OWS P results in the coupling effectively decreasing by one third, and about a 10% decrease in the damping. That is, while multiplicative noise acts to increase the persistence of SST and AIRT anomalies by reducing the damping of each, it acts to reduce persistence by weakening the coupling. At OWS P this also means that multiplicative noise roughly doubles AIRT variability on subseasonal timescales.

Additionally, the classical stochastic view with additive noise implies that sea surface temperature (SST) and air temperature (AIRT) anomalies obey a Gaussian distribution. However, OWS observations reveal that joint probability distribution functions (PDFs) of daily averaged SST and AIRT anomalies are actually significantly non-Gaussian. It is shown that it is the state-dependent character of the rapidly-varying boundary-layer heat fluxes that appears to be responsible for the observed non-Gaussianity of SST and AIRT anomalies. It is concluded that the effect of state-dependent noise is crucial to understand and model atmosphere-ocean coupling.

# 1 Introduction

Coupling of the atmosphere and ocean is largely due to fluxes of momentum and heat through their common boundary, the sea surface. Most of this exchange during the cold season is a result of the turbulent nature of the atmospheric boundary layer; molecular momentum and heat transfers play a very minor role in atmosphere-ocean coupling. That is, in an hypothetical world without any sea surface winds there would be little air-sea interaction. Wind-driven turbulent heat fluxes are commonly expressed in terms of simple bulk formulae that depend upon the strength of the wind speed  $|\mathbf{U}|$  and typically take the form  $f = \beta(T_o - T_a)|\mathbf{U}|$  (positive flux upward), where  $T_o$  and  $T_a$  are the sea surface temperatures (SST) and air temperatures (AIRT) respectively, and  $\beta$  is a parameter including all the constants like densities of sea-water and air, specific heats, the Bowen ratio and so forth [see Frankignoul and Hasselmann (1977), Sura et al. (2006) or any textbook on air-sea interaction, e.g. Kraus and Businger (1994), for details]. Variations in this flux can then drive variations in midlatitude SST and AIRT.

Barsugli and Battisti (1998) (hereafter BB98) proposed a simple heuristic model to gauge the effect of atmosphere-ocean thermal coupling, consisting of two linear equations for the rate of change of the AIRT anomaly  $T'_a$  and the SST anomaly  $T'_o$ :

$$\begin{aligned}\frac{dT'_a}{dt} &= -aT'_a + bT'_o + \eta_a , \\ \frac{dT'_o}{dt} &= cT'_a - dT'_o ,\end{aligned}\tag{1}$$

where  $\eta_a$  is Gaussian white-noise representing rapidly varying weather fluctuations, the parameters  $a$  and  $d$  are the damping coefficients, and  $b$  and  $c$  are coupling coefficients. How

surface winds drive surface fluxes was not explicitly considered in this model; rather, BB98 suggest that the coupling coefficients represent the effects of both feedback due to surface fluxes (turbulent and radiative) and the dynamical response of the atmosphere to SST anomalies. The main point of BB98 is that the atmosphere-ocean coupling enhances the variance in both media due to a decreased energy flux between the atmosphere and the ocean, which they termed reduced thermal damping.

BB98 can be viewed as a straightforward extension of the Frankignoul and Hasselmann (1977) null hypothesis for SST variability, where the effect of atmospheric forcing on SST anomalies is commonly represented by a simple stochastic model of the oceanic mixed-layer,

$$\frac{dT'_o}{dt} = -\lambda T'_o + \eta \quad , \quad (2)$$

(Hasselmann 1976; Frankignoul and Hasselmann 1977, hereafter FH77). Here,  $\lambda$  is a constant rate coefficient representing the transfer of heat from the slowly evolving mixed-layer heat anomaly and  $\eta$  is Gaussian white-noise representing heat fluxes due to rapidly varying weather fluctuations. However, (Sura et al. 2006, hereafter SNA) showed that this implicitly ignores the effect of rapid variations in the wind speed,  $|\mathbf{U}|'$ , upon  $\lambda$ . That is, because surface wind speeds have relatively large variability and are almost uncorrelated from day to day, it is more appropriate (as derived from first principles) to parameterize  $\lambda$  as a stochastic process in which  $\lambda = \bar{\lambda} + \lambda'$ , where  $\bar{\lambda}$  is constant but  $\lambda'$  is white noise. This results in the addition of a state-dependent, or multiplicative, noise  $-\lambda'T'_o$  to (2) that depends upon the SST anomaly itself. As described in SNA, such multiplicative noise has two notable consequences. First, it results in non-Gaussian probability distribution functions (PDFs) for SSTs, even though the deterministic portion of (2) remains linear. Second, it increases

the autocorrelation time scale of SST anomalies, even as the autocorrelation function remains exponential as suggested by FH77. This is a consequence of a phenomenon known as noise-induced drift, which occurs because the time mean of the multiplicative noise term,  $\overline{\lambda' T'_o}$ , is not zero even though the time mean of  $\lambda'$  is zero. SNA showed that their simple modification of the FH77 paradigm compared well to observed non-Gaussian PDFs of daily SSTs at several Ocean Weather Stations (OWSs), and using a single-column mixed-layer ocean model suggested that most of the multiplicative noise during winter was likely due to rapidly varying sensible and latent heat flux anomalies.

The success of the SNA multiplicative noise approach for uncoupled SST variability immediately suggests that it is also relevant to the coupled problem, particularly as formulated by BB98. For example, note that anomalies of the heat flux  $f$  can be partitioned into three terms that depend upon the products  $|\overline{\mathbf{U}}|(T'_o - T'_a)$ ,  $|\mathbf{U}'|(\overline{T_o} - \overline{T_a})$ , and  $|\mathbf{U}'|(T'_o - T'_a)$ , respectively. While the first term represents deterministic coupling of AIRT and SST anomalies, and the second term is a source of additive (that is, state-independent) noise, the third term potentially represents multiplicative noise. Thus, we might expect that in the BB98 model, the damping and coupling coefficients  $(a, b, c, d)$  should all contain stochastic components. As in SNA, noise-induced drift due to fluctuations in the damping coefficients would act to increase persistence of both SST and AIRT anomalies. However, as noted in a different climate context by Sardeshmukh et al. (2001), noise-induced drift due to fluctuations in the coupling coefficients might act to *decrease* persistence, acting to oppose reduced thermal damping. Moreover, stochastic fluctuations in the coupling parameters raises the possibility that not all air-sea coupling is necessarily deterministic, as is assumed in the BB98 framework.

In this paper we study the impact of rapidly-varying (on daily and sub-daily timescales) sea surface heat fluxes on atmosphere-ocean coupling by deriving a simple coupled model of SST and AIRT variability that accounts for rapid fluctuations in surface heat fluxes due to rapid variability of sea surface winds. The outline of the paper is as follows. First, because the effect of the noise-induced drift is crucial to understand the effect of fluctuation winds on atmosphere-ocean coupling, we will explain the physical cause of the noise-induced drift in a heuristic way (section 2). The coupled model, similar in form to BB98 except that it includes multiplicative noise, is then introduced in section 3. In section 4 we use inverse methods to estimate the parameters of our simple coupled model, show that the non-Gaussianity of anomalous SST variability during the extended winter season at several Ocean Weather Stations can be reproduced by this model, and discuss the impact of the multiplicative noise on the power spectra of SST and AIRT. Finally, section 5 provides a summary and discussion.

## 2 Noise-induced drift

To understand how multiplicative noise affects atmosphere-ocean coupling, it is useful to first clarify how introducing stochastic components with zero mean to the coefficients in (1) can nevertheless result in a noise-induced drift. Readers already familiar with stochastic differential equations may wish to skip this section, whereas those interested in rigorous math should consult one of the available textbooks (e.g., Gardiner 2004; Horsthemke and Léfever 1984; Kloeden and Platen 1992; Paul and Baschnagel 1999). However, to heuristically

explain the noise-induced drift consider the simplified system

$$\frac{d\delta T}{dt} = -\delta T(\overline{|\mathbf{U}|} + |\mathbf{U}'|), \quad (3)$$

where wind-driven heat flux acts to relax (damp) the air-sea temperature difference  $\delta T \equiv (T'_a - T'_o)$  towards its climatological (equilibrium) state  $\delta T = 0$ . The relaxation is forced by two components of the heat flux, one driven by the mean wind speed  $\overline{|\mathbf{U}|}$  and the other driven by the anomalous wind speed  $|\mathbf{U}'|$ .

For some climate problems of interest, stochastic differential equations are obtained by approximating rapidly varying quantities as white-noise. In (3), for example, note that  $\delta T$  evolves much more slowly than  $|\mathbf{U}'|$ , so  $|\mathbf{U}'|$  is approximated by white-noise. In reality, of course, the underlying systems are usually continuous in both time and space. That is, physical processes are often smooth with at least a small degree of autocorrelation. In our specific example this means that there exists a small time increment during which the anomalous wind  $|\mathbf{U}'|$  can be considered constant (loosely speaking, this is the definition of a Stratonovich system; see the appendix for a more detailed discussion).

Consider an ensemble of realizations for which it is assumed that the wind anomaly forcing is Gaussian with a standard deviation  $\sigma_{|\mathbf{U}'|}$  and PDF  $p(|\mathbf{U}'|) \equiv p$ . We ask how the ensemble mean trajectory that passes through the point  $\delta T_0$  evolves for a small time increment where the wind speed anomaly  $|\mathbf{U}'|$  is approximately constant. To answer, note that the average response to positive  $+|\mathbf{U}'|$  and negative  $-|\mathbf{U}'|$  kicks is

$$\langle \delta T_+ \rangle = \delta T_0 \int_0^\infty \exp\left(-(\overline{|\mathbf{U}|} + |\mathbf{U}'|p)t\right) d|\mathbf{U}'| \quad , \quad (4)$$

$$\langle \delta T_- \rangle = \delta T_0 \int_0^\infty \exp\left(-(\overline{|\mathbf{U}|} - |\mathbf{U}'|p)t\right) d|\mathbf{U}'| \quad . \quad (5)$$



Therefore, the mean response to the anomalous wind forcing will be

$$\langle \delta T \rangle = \delta T_0 \frac{1}{2} \int_0^\infty \left[ \exp \left( -(|\overline{\mathbf{U}}| + |\mathbf{U}'|p)t \right) + \exp \left( -(|\overline{\mathbf{U}}| - |\mathbf{U}'|p)t \right) \right] d|\mathbf{U}'| \quad . \quad (6)$$

However, without wind forcing anomalies  $\langle \delta T \rangle$  evolves according to

$$\langle \delta T \rangle = \delta T_0 \exp \left( -|\overline{\mathbf{U}}|t \right) \quad . \quad (7)$$

Note that in the presence of anomalous wind speeds  $|\mathbf{U}'|$  the average response  $\langle \delta T \rangle$  differs from the corresponding response without the anomalous forcing. That is, in Eq. (6) the responses to anomalous kicks of opposite signs (say,  $|\mathbf{U}'| = +\delta|\mathbf{U}'|$  and  $|\mathbf{U}'| = -\delta|\mathbf{U}'|$ ) do not cancel each other. This results in a noise-induced drift: while the wind anomalies with opposite signs cancel each other, the responses do not average to zero. The situation is depicted in Fig. 1, where we plotted Eq. (6) (response with anomalous wind speeds; solid line) and Eq. (7) (response without anomalous wind speeds; dashed line) for  $|\overline{\mathbf{U}}| \equiv 1$ ,  $\sigma_{|\mathbf{U}'|} \equiv 0.5$ , and  $\delta T_0 \equiv 1$ . It can be seen that with anomalous wind speeds  $\langle \delta T \rangle$  decays slightly slower than in the case with the mean wind only. That is, in this particular case (linear system with linear multiplicative noise) the stochastic forcing effectively reduces the damping of the system. In more general terms, the stochastic forcing induces an additional drift term, the noise-induced drift. Note that this drift, although due to a stochastic (i.e., unpredictable) term, results in a predictable change to the evolution of  $\langle \delta T \rangle$ .

### 3 A Simple coupled model

In this section we show that rapid surface wind variability needs to be considered in any complete model of atmosphere-ocean interactions, and that its effects can be modeled with

multiplicative noise. Our approach is to extend the BB98 stochastic model by explicitly considering wind induced heat flux anomalies in the context of a simple mixed-layer model. Recall that BB98 assumed their 1-D model included the dynamical atmospheric response to SST anomalies and their  $b$  is assumed to include the effect of this response. In the following model derivation, coupling is due to only surface heat flux, so that all model parameters are based on local coupling alone. However, in section 4 model parameters will be determined from observations and thus might also include the effects of atmospheric dynamics.

### 3.1 The basic equations

Consider a one-dimensional thermodynamic model for the upper mixed-layer ocean coupled to a one-dimensional mixed-layer atmosphere. That is, in the atmosphere we assume a well mixed and horizontally homogeneous layer with temperature  $T_a$  with effective heat capacity  $\gamma_a$  in contact with the underlying ocean. In the ocean we assume a well mixed and horizontally homogeneous layer of constant depth and temperature  $T_o$  with effective heat capacity  $\gamma_o$  in contact with the overlying atmosphere. For the sake of simplicity, all effects of horizontal advection, humidity, and salinity are ignored. Then local heat budget equations for AIRT  $T_a$  and SST  $T_o$  can be written as

$$\gamma_a \frac{dT_a}{dt} = f(T_a, T_o, |\mathbf{U}|) - \lambda_a T_a + \xi_a \equiv F_a(T_a, T_o, |\mathbf{U}|) + \xi_a \quad , \quad (8)$$

$$\gamma_o \frac{dT_o}{dt} = -f(T_a, T_o, |\mathbf{U}|) - \lambda_o T_o + \xi_o \equiv F_o(T_a, T_o, |\mathbf{U}|) + \xi_o \quad , \quad (9)$$

where  $f(T_a, T_o, |\mathbf{U}|)$  is the latent and sensible heat flux through the air-sea interface which depends on the SST  $T_o$ , air temperature  $T_a$ , and wind speed  $|\mathbf{U}|$ . External forcings of  $T_a$

and  $T_o$  (that is, forcing not due explicitly to air-sea interaction) are represented by  $\xi_a$  and  $\xi_o$ , respectively. The remaining terms  $-\lambda_a T_a$  and  $-\lambda_o T_o$  represent the effective damping of each component. The bulk formula for the heat flux  $f$  is

$$f = \beta(T_o - T_a)|\mathbf{U}| \ , \quad (10)$$

where  $\beta$  is a constant parameter that depends upon the bulk transfer coefficients and the inverse Bowen ratio (ratio of latent to sensible heat flux). That is, we assume that the heat flux variability is only due to AIRT, SST, and wind speed variability. This is a reasonable approximation in our simple framework, since cold season heat flux anomalies are strongly related to wind speed anomalies (e.g., Ronca and Battisti 1997; Alexander and Scott 1997). It is not a perfect approximation, of course. In particular, there are two key simplifying assumptions in the bulk heat flux formula (10). First, the sensible and latent heat fluxes are combined together into one term (as, e.g., Frankignoul and Hasselmann 1977), which also removes the explicit dependence of the latter on humidity. Second,  $\beta$  is set to a constant, whereas in reality  $\beta$  depends somewhat on the air-sea temperature difference  $T_o - T_a$  (the stability) and the strength of the wind speed  $|\mathbf{U}|$  (e.g., Large and Pond 1982). On the other hand, SNA found for the uncoupled problem that while these simplifications can affect the results quantitatively, they do not obscure the main points that are drawn from the derivation using (10); that is, the fundamental nature of multiplicative noise can be established from Eq. (10) since it contains the basic physics involved in local air-sea coupling.

### 3.2 Taylor expansion

For small temperature anomalies  $T'_a$  and  $T'_o$  a Taylor expansion of  $F_a$  and  $F_o$  with respect to

$T_a = \bar{T}_a + T'_a$  and  $T_o = \bar{T}_o + T'_o$  yields

$$\gamma_a \frac{d(\bar{T}_a + T'_a)}{dt} = F_a(\bar{T}_a, \bar{T}_o, |\mathbf{U}|) + \frac{\partial F_a}{\partial T_a} T'_a + \frac{\partial F_a}{\partial T_o} T'_o + \bar{\xi}_a + \xi'_a , \quad (11)$$

$$\gamma_o \frac{d(\bar{T}_o + T'_o)}{dt} = F_o(\bar{T}_a, \bar{T}_o, |\mathbf{U}|) + \frac{\partial F_o}{\partial T_a} T'_a + \frac{\partial F_o}{\partial T_o} T'_o + \bar{\xi}_o + \xi'_o , \quad (12)$$

where ( $T_a$ -dimension)

$$\begin{aligned} F_a(\bar{T}_a, \bar{T}_o, |\mathbf{U}|) &= \beta(\bar{T}_o - \bar{T}_a)|\mathbf{U}| - \lambda_a \bar{T}_a , \\ \frac{\partial F_a}{\partial T_a} T'_a &= (-\beta|\mathbf{U}| - \lambda_a) T'_a , \\ \frac{\partial F_a}{\partial T_o} T'_o &= \beta|\mathbf{U}| T'_o , \end{aligned} \quad (13)$$

and ( $T_o$ -dimension)

$$\begin{aligned} F_o(\bar{T}_a, \bar{T}_o, |\mathbf{U}|) &= -\beta(\bar{T}_o - \bar{T}_a)|\mathbf{U}| - \lambda_o \bar{T}_o , \\ \frac{\partial F_o}{\partial T_a} T'_a &= \beta|\mathbf{U}| T'_a , \\ \frac{\partial F_o}{\partial T_o} T'_o &= (-\beta|\mathbf{U}| - \lambda_o) T'_o . \end{aligned} \quad (14)$$

In the following we use the standard assumption that the evolution of the time mean temperatures  $\bar{T}_a$  and  $\bar{T}_o$  is balanced by the time mean of the RHS of Eqs. (11) and (12). Note that, as are all other quantities, the wind speed is split into a mean ( $|\bar{\mathbf{U}}|$ ) and a deviation from the mean ( $|\mathbf{U}'|$ ).

This results in the coupled set of equations for the anomalous temperatures  $T'_a$  and  $T'_o$ :

$$\gamma_a \frac{dT'_a}{dt} = \beta(\bar{T}_o - \bar{T}_a)|\mathbf{U}'| - [\beta(|\bar{\mathbf{U}}| + |\mathbf{U}'|) + \lambda_a] T'_a + \beta(|\bar{\mathbf{U}}| + |\mathbf{U}'|) T'_o + \xi'_a , \quad (15)$$

$$\gamma_o \frac{dT_o'}{dt} = -\beta(\overline{T_o} - \overline{T_a})|\mathbf{U}|' + \beta(\overline{|\mathbf{U}|} + |\mathbf{U}|')T_a' - [\beta(\overline{|\mathbf{U}|} + |\mathbf{U}|') + \lambda_o] T_o' + \xi_o' . \quad (16)$$

Apart from external forcing, which for simplicity from this point will be assumed to be additive noise, perturbations in this simple model are only due to wind induced heat flux anomalies. The effect of the wind on anomalous SST and AIRT variability is split into two parts, due to the mean and the anomalous wind speeds. Typically, the ratio of daily averages of mean wind speed and the anomalous wind speed is only about 2 to 1 (e.g., Monahan 2006b). As a consequence, a scale analysis of the flux terms (SNA) shows that the terms depending upon  $|\mathbf{U}|'$  are too large to ignore.

We now assume that the wind speed anomalies  $|\mathbf{U}|'$  can be approximated by Gaussian white-noise. This assumption is justifiable if daily wind speed anomalies vary much more rapidly than SST and AIRT and if they have a distribution that is nearly Gaussian. For example, at OWS P wind speed anomalies have a 1-day autocorrelation time-scale (Fig. 2a) and deviations from Gaussianity are relatively small (Fig. 2b). Both SST and AIRT are uncorrelated with wind speed, but while SST has an autocorrelation time-scale of a few months, AIRT has only about a 3.5-day autocorrelation time-scale. Thus, while approximating wind speed as white-noise is an excellent approximation compared to SST, it may be only a fair approximation compared to AIRT. Perhaps a better approximation of wind speed is to represent it as red-noise with a short decorrelation time, or more simply scaling the white-noise amplitude by the square root of the wind speed time-scale. In either case this correction may be relatively minor, as suggested by a previous analysis of multiplicative red-noise in a system similar to that discussed below (Sardeshmukh et al. 2003).

Employing the Gaussian white-noise assumption for wind speed, Eqs. (15) and (16)

become stochastic differential equations (SDEs) that in matrix form are expressed as

$$\frac{d\mathbf{T}'}{dt} = \mathbf{A}\mathbf{T}' + \mathbf{B}_M(\mathbf{T}')\boldsymbol{\eta}_M + \mathbf{B}_A\boldsymbol{\eta}_A, \quad (17)$$

with the  $2 \times 2$  matrices

$$\mathbf{A} = \begin{pmatrix} -(\frac{\beta}{\gamma_a}|\overline{\mathbf{U}}| + \frac{\lambda_a}{\gamma_a}) & \frac{\beta}{\gamma_a}|\overline{\mathbf{U}}| \\ \frac{\beta}{\gamma_o}|\overline{\mathbf{U}}| & -(\frac{\beta}{\gamma_o}|\overline{\mathbf{U}}| + \frac{\lambda_o}{\gamma_o}) \end{pmatrix} \equiv \begin{pmatrix} -a & b \\ c & -d \end{pmatrix} \quad (18)$$

and

$$\mathbf{B}_M(\mathbf{T}') = \begin{pmatrix} \frac{\sigma^M \beta}{\gamma_a}(T'_o - T'_a + \Pi) & 0 \\ \frac{\sigma^M \beta}{\gamma_o}(T'_a - T'_o - \Pi) & 0 \end{pmatrix} \equiv \begin{pmatrix} B_{11}(T'_a, T'_o) & 0 \\ B_{21}(T'_a, T'_o) & 0 \end{pmatrix}. \quad (19)$$

The state vector is  $\mathbf{T}' = (T'_a, T'_o)^T$ , and the mean SST-AIRT temperature difference is  $\Pi = \overline{T_o} - \overline{T_a}$ . There are two sources of noise: the additive stochastic noise vector  $\boldsymbol{\eta}_A = (\eta_{T_a}^A, \eta_{T_o}^A)^T$  multiplied by an amplitude matrix  $\mathbf{B}_A$  that can have non-zero off-diagonal elements, and multiplicative noise resulting from the matrix  $\mathbf{B}_M(\mathbf{T}')$  multiplied by the noise vector  $\boldsymbol{\eta}_M = (\eta_{T_a}^M, 0)^T$ . The variance of the wind speed anomaly  $|\mathbf{U}'|$  is absorbed in the constant  $\sigma^M$  [in the manner discussed by Sardeshmukh et al. (2001)] so that  $\eta_{T_a}^M$  has unit variance. The stochastic components are assumed to be independent, normalized Gaussian white-noise processes.

Without multiplicative noise ( $\mathbf{B}_M \equiv 0$ ), Eq. (17) is structurally equivalent to the system used by BB98, although they assumed no white-noise forcing of SST. With multiplicative noise, the system can result in non-Gaussian PDFs of  $\mathbf{T}'$ . However, if  $\Pi \equiv 0$ , this system is symmetric and will only result in symmetric distributions. That is, the  $\Pi \eta_{T_a}^M$  terms in  $\mathbf{B}_M(\mathbf{T}')$ , which represent additive noise that is correlated with the multiplicative noise, induce skewness.

Because we approximated a rapidly fluctuating quantity ( $|\mathbf{U}'|$ ) with a small but finite

correlation time as white-noise, Eq. (17) has to be treated as a Stratonovich SDE (see the appendix for a brief discussion of SDEs and stochastic dynamics). The corresponding noise-induced drift [see Eq. (A.3)] is then

$$\frac{1}{2} \left( \frac{\partial B_{11}}{\partial T'_a} \right) B_{11} + \frac{1}{2} \left( \frac{\partial B_{11}}{\partial T'_o} \right) B_{21} = -\frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a^2} (T'_o - T'_a + \Pi) - \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a \gamma_o} (T'_o - T'_a + \Pi) \quad (20)$$

for the  $T'_a$ -component, and

$$\frac{1}{2} \left( \frac{\partial B_{21}}{\partial T'_a} \right) B_{11} + \frac{1}{2} \left( \frac{\partial B_{21}}{\partial T'_o} \right) B_{21} = \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a \gamma_o} (T'_o - T'_a + \Pi) + \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_o^2} (T'_o - T'_a + \Pi) \quad (21)$$

for the  $T'_o$ -component. If we use the fact that the atmospheric heat capacity is much smaller than the oceanic one (that is,  $\gamma_a \ll \gamma_o$ ) the second terms on the right hand sides can be neglected:

$$\frac{1}{2} \left( \frac{\partial B_{11}}{\partial T'_a} \right) B_{11} + \frac{1}{2} \left( \frac{\partial B_{11}}{\partial T'_o} \right) B_{21} \approx -\frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a^2} (T'_o - T'_a + \Pi) \quad (22)$$

for the  $T'_a$ -component, and

$$\frac{1}{2} \left( \frac{\partial B_{21}}{\partial T'_a} \right) B_{11} + \frac{1}{2} \left( \frac{\partial B_{21}}{\partial T'_o} \right) B_{21} \approx \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a \gamma_o} (T'_o - T'_a + \Pi) \quad (23)$$

for the  $T'_o$ -component.

A summary figure of our model is shown in Fig. 3, illustrating the distinction between *air-sea interaction with mean wind only* (upper panel) and *air-sea interaction with mean and fluctuating winds* (lower panel). The physical set-ups are shown on the left hand sides of each panel, while the equivalent mathematical relations are depicted on the panel's right hand sides. In both cases (upper and lower panels) the oceanic and atmospheric temperatures are damped by radiative effects  $-\lambda_a T_a$  and  $-\lambda_o T_o$ . When only the mean wind drives heat flux (upper panel), the heat flux coupling is  $\pm \beta (T_o - T_a) |\overline{\mathbf{U}}|$ , and the evolution of SST and

AIRT is given by the deterministic drift matrix  $\mathbf{A}$  with parameters  $-a, b, c, -d$ . This is depicted by the corresponding arrows:  $-a$  damps the atmosphere,  $b$  is the oceanic forcing of the atmosphere,  $c$  is the atmospheric forcing of the ocean, and  $-d$  damps the ocean. In the case where the heat flux is due to both the mean and fluctuating winds, the predictable evolution of atmosphere-ocean temperatures is now given by the *effective* drift matrix

$$\tilde{\mathbf{A}} = \begin{pmatrix} -a + \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a^2} & b - \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a^2} \\ c - \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a \gamma_o} & -d + \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a \gamma_o} \end{pmatrix} \equiv \begin{pmatrix} -\tilde{a} & \tilde{b} \\ \tilde{c} & -\tilde{d} \end{pmatrix}. \quad (24)$$

That is, the multiplicative noise results in a noise-induced drift [see Eqs. (22) and (23)] that effectively decreases the amplitudes of the damping and coupling parameters, altering the predictable dynamics of air-sea interaction. In particular, note that multiplicative noise can lead to effectively reduced damping through its effect on  $\tilde{a}$  and  $\tilde{d}$ , yet simultaneously effectively *weaken* air-sea coupling and thus weaken the coupling-induced reduced thermal damping proposed by BB98.

## 4 Parameter estimation from observations

To gauge the effect of multiplicative noise on atmosphere-ocean coupling, we next estimate the parameters of the simple coupled model (17) from Ocean Weather Station (OWS) data. Of all the Ocean Weather Stations, OWS P may be best suited for this purpose (Hall and Manabe 1997; Sura et al. 2006). It has a long high-quality record, the El Niño-Southern Oscillation (ENSO) signal is relatively weak there (e.g., Alexander et al. 2002), and it is located



far from strong currents. We will, therefore, in the following provide a detailed discussion of OWS P (see Table 1) as our “prototype” for midlatitude, wintertime atmosphere-ocean coupling. A brief discussion of results from shorter-record stations is presented at the end of this section.

## 4.1 Data

Daily SST and AIRT anomalies were determined as follows. First, daily averages were calculated from the raw 3-hourly data. Then the climatological monthly averages were estimated. A daily climatology was constructed by linear interpolation using these monthly averages as base points. Finally, daily anomalies were calculated by subtracting the daily climatology from the mean daily values.

SNA have already shown that SST variability is best described by a stochastic model of surface heat fluxes during the extended wintertime (November-April), since the anomalous mixed-layer temperature (SST) tendency is predominantly due to the net turbulent heat flux anomalies and not to changes in the mixed-layer depth (Alexander and Penland 1996; Sura et al. 2006). Also, radiative flux anomalies have only a minor impact during the cold season (Park et al. 2005; Frankignoul and Kestenare 2002). Therefore, the following analysis is restricted to the extended winter season.

## 4.2 Parameter estimation

First, the direct finite-difference technique (see appendix) is used to estimate the effective drift of the coupled system (AIRT and SST) at OWS P; results are shown in Figs. 4a and 4b. This technique can determine either linear or non-linear drifts. However, we find that the drift is nearly linear, suggesting that the linear approximation of Eqs. (17) and (24) [and earlier used by Barsugli and Battisti (1998)] is a good approximation of the real system.

For OWS P the linear effective drift matrix is:

$$\tilde{\mathbf{A}} = \begin{pmatrix} -0.35 & 0.22 \\ 0.01 & -0.04 \end{pmatrix}. \quad (25)$$

The evolution of  $T'_a$  is strongly damped ( $-\tilde{a}$ ) and weakly coupled to  $T'_o$  ( $\tilde{b}$ ). In the same way the evolution of  $T'_o$  is weakly coupled to  $T'_a$  ( $\tilde{c}$ ) and damped ( $-\tilde{d}$ ).

The method used to estimate the multiplicative noise matrix  $\mathbf{BB}^T$  is similar to that used to estimate the drift. However, as noted in the appendix, estimation of the multiplicative noise matrix  $\mathbf{BB}^T$  is prone to error. The most practical way to detect the systematic error made by using a finite time step is to change  $\Delta t$  and to compare the results. Here the method yields stable results apart from an overall constant, implying that the finite-difference estimate of  $\mathbf{BB}^T$  does not depend sensitively on the time step  $\Delta t$ . Unfortunately, because the components of  $\mathbf{BB}^T$  vary over more than two orders of magnitude, only the largest component  $(\mathbf{BB}^T)_{11}$ , shown in Fig. 4c, can be determined with confidence. Note that this term includes both additive and multiplicative noise.

Recall that in the simple model,  $(\mathbf{BB}^T)_{11} = (B_{11})^2$  [cf. Eq. (19)]. The observed  $(\mathbf{BB}^T)_{11}$  has a structure quite similar to  $(B_{11})^2$ , with a pronounced minimum for  $T'_o - T'_a \approx -0.7K = \Pi$

and a roughly quadratic increase orthogonally away from the minimum line. It is thus reasonable to use  $(\mathbf{B}\mathbf{B}^T)_{11}$  to estimate the parameters of the linear multiplicative noise terms of the simple coupled system. First, given Eq. (19)  $(\sigma^M\beta/\gamma_a)^2$  can be estimated from Fig. 4c. Next, the ratio  $\gamma_a/\gamma_o$  is given by the ratio  $\tilde{c}/\tilde{b}$  [see Eq. (24)]. Given  $(\sigma^M\beta/\gamma_a)^2$  the ratio  $\gamma_a/\gamma_o$  is then used to calculate  $(\sigma^M\beta/\sqrt{\gamma_a\gamma_o})^2$  and  $(\sigma^M\beta/\gamma_o)^2$ . This procedure results in the following estimates:

$$\gamma_a/\gamma_o \approx 0.05 , \quad (26)$$

$$(\sigma^M\beta/\gamma_a)^2 \approx 0.2 , \quad (27)$$

$$(\sigma^M\beta/\sqrt{\gamma_a\gamma_o})^2 \approx 0.01 , \quad (28)$$

$$(\sigma^M\beta/\gamma_o)^2 \approx 0.0005 . \quad (29)$$

$\gamma_a/\gamma_o$  determined in this manner is identical to that used by BB98, which represents the ratio of the heat capacities of a tropospheric column and wintertime oceanic mixed-layer. The other parameters are consistent with typical values of  $\beta$  and wind speed variability at OWS P. The covariance of the multiplicative noise is then

$$\langle \mathbf{B}_M \mathbf{B}_M^T \rangle = \begin{pmatrix} 0.48 & -0.024 \\ -0.024 & 0.0012 \end{pmatrix} . \quad (30)$$

Given the multiplicative noise, the additive noise required to reproduce the observed covariance structure can now be calculated from the fluctuation-dissipation relation (see appendix):

$$\mathbf{B}_A \mathbf{B}_A^T = -\tilde{\mathbf{A}} \mathbf{C}_0 - \mathbf{C}_0 \tilde{\mathbf{A}} - \langle \mathbf{B}_M \mathbf{B}_M^T \rangle , \quad (31)$$

where  $\mathbf{C}_0$  is the data's covariance matrix at lag zero, yielding

$$\mathbf{B}_A \mathbf{B}_A^T = \begin{pmatrix} 0.92 & 0.07 \\ 0.07 & 0.03 \end{pmatrix}. \quad (32)$$

Note that the multiplicative noise comprises about one third of the overall atmospheric noise forcing but is an almost negligible component of SST noise forcing. Also, while the pure additive noises of SST and AIRT have a 0.4 correlation, SST and AIRT multiplicative noises are anticorrelated, so that overall SST and AIRT noises have only a 0.2 correlation.

Having determined the noise-induced drift, the deterministic drift is calculated as:

$$\mathbf{A} = \begin{pmatrix} -0.45 & 0.32 \\ 0.015 & -0.045 \end{pmatrix}. \quad (33)$$

Comparison of  $\mathbf{A}$  and  $\tilde{\mathbf{A}}$  suggests that the noise-induced drift reduces the effective coupling parameters by roughly one third, and decreases the effective damping by roughly 10% for SST and 20% for TAIR.

### 4.3 Testing the multiplicative white-noise model

We next test if the two assumptions of linear dynamics and the white-noise approximation made in Eq. (17) are consistent with the observations at OWS P. For any observation  $\mathbf{T}'_{obs}(t)$  the effective drift  $\tilde{\mathbf{A}}\mathbf{T}'_{obs}(t)$  estimated from data can be used to calculate a forward time step  $\Delta t$ :  $\mathbf{T}'(t + \Delta t) = \tilde{\mathbf{A}}\mathbf{T}'_{obs}(t)\Delta t + \mathbf{T}'_{obs}(t)$ . If the white-noise assumption is correct the residual

$$\mathbf{r} \equiv \mathbf{T}'_{obs}(t + \Delta t) - \mathbf{T}'(t + \Delta t) \quad (34)$$

should equal the white-noise terms. Autocorrelation functions and PDFs of the residual are shown in Fig. 5. For both AIRT and SST anomalies the autocorrelations are close to

zero after one time step of 1 day. That is, the residuals are practically uncorrelated on the resolved time scale, consistent with the white-noise approximation. Moreover, the PDFs of the residuals are highly non-Gaussian, indicating that multiplicative white-noise is essential to represent the noise in the coupled system.

Not only are the residuals uncorrelated after a day, but there is also no significant long range correlation in Fig. 5a. This confirms that the linear model of local air-sea interaction is a good approximation of the observed coupled thermodynamics at OWS P; that is, the evolution of AIRT and SST is captured with the simple linear model. Note, however, that the assumption of additive Gaussian white-noise made in the BB98 model (and also, e.g., Mosedale et al. 2005) appears invalid.

## 4.4 Probability Density Functions

In the previous subsection 4.3 we have seen that the simple coupled model reproduces the deterministic dynamics of the observations rather well. As a hard test, we next examine if the multiplicative noise model reproduces the observed joint PDF of AIRT and SST anomalies at OWS P. PDFs are a useful measure to examine the dynamics of stochastic systems. In particular, deviations from Gaussianity, or anomalous statistics, can shed light on the underlying dynamics (e.g., Peinke et al. 2004; Sura et al. 2005). To calculate the joint PDFs of AIRT and SST anomalies we use a parametric method where the parameters of the distribution are determined by a Maximum Likelihood Estimate (MLE). The parametric distribution we use is the skew- $t$ , a skewed and kurtosed alternative to the normal distribution which is capable of adapting very closely to skewed and heavy-tailed data (Azzalini and

Capitanio 2003; Jones and Faddy 2003; Azzalini 2005; R Development Core Team 2004). SNA showed that the skew- $t$  distribution does an excellent job describing the PDF of SST anomalies by comparing a simple histogram to the skew- $t$  estimate. We did the same for the joint PDFs and found that the skew- $t$  distribution captures the structure of the histograms very well.

The joint PDF anomalies (deviations from Gaussianity) of our simple model are shown in Fig. 6a, whereas the observed joint PDF anomalies are shown in Fig. 6b. Here we normalized the anomaly timeseries to have zero means and unit variances to make the comparison of the PDFs easier to view.

The basic non-Gaussian structures and their amplitudes are the same in both the model and the observations. The joint PDF is stronger than Gaussian around the origin, has weak flanks on both sides of the peak (weaker for negative AIRT anomalies than for positive ones), and has heavy tails (not well seen here because of the contour levels used). The maxima of both the modeled and observed PDF anomalies lie shifted into the positive AIRT-plane, consistent with the effect of the mean air-sea temperature difference in the multiplicative noise. These results are reasonably robust, to factor of two variations in the parameters (26-29). The main difference between the model and observations is in the positive AIRT half-plane, where there is a slight mismatch of the PDF anomaly's orientation and amplitude. This mismatch may be due to the simplicity of the model; for example, the stability dependence of the bulk flux parameter  $\beta$  was neglected, but in reality the value should be different for  $T_a > T_o$  and  $T_a < T_o$  (e.g., Large and Pond 1982, see also the discussion of the model results in SNA). Still, given the simplicity of the model the agreement

of the simulated and observed joint PDFs in Fig. 6 is remarkably good. The most important point is that the effective deterministic drift of our model is linear, and that parameterizing state-dependent rapidly-varying heat flux with multiplicative noise can explain the observed non-Gaussian statistics.

## 4.5 Spectra

The impact of multiplicative noise upon the variability of AIRT and SST anomalies is demonstrated by numerically integrating two different coupled stochastic models, with and without multiplicative noise. To do so, we integrate (17) forwards for  $10^6$  days (the “multiplicative noise model”), and then repeat the calculations by removing the multiplicative noise term (e.g.,  $\mathbf{B}_M \equiv 0$ ; the “additive noise model”). The crucial difference is that while both model runs have the same deterministic dynamics, the additive noise model does not have the noise-induced drift. In addition, the overall variance of modeled anomalous AIRT and SST variability is reduced in the additive noise model because the strength of the noise forcing itself is reduced. More precisely, the observed covariance structure of AIRT and SST variability is not conserved in the additive noise model. This experiment visualizes the bias one would get if a coupled model were forced with winds that have no variability at all. Note that the additive noise model does not actually contain *all* the additive noise, since the portion of the additive noise due to wind variability is also removed.

The spectra for this set of model runs are shown in Fig. 7, for AIRT (top) and SST (bottom). The spectra of the additive noise model are indicated by the dashed lines, and the spectra of the full multiplicative noise model are indicated by the solid lines. Consistent with

the amplitude of the noise-induced drift found in section 4.2, the removal of the multiplicative noise has a much larger impact on the AIRT spectrum than the SST spectrum. AIRT variance decreases by almost 40%, with most of this impact evident on the subseasonal timescale, where variance decreases by half and the peak in the spectrum shifts to shorter periods. Conversely, the effect of multiplicative noise is much less on periods of months to years. That is, the multiplicative noise has an impact not just on the overall variance but on the shape of the power spectrum, leading to relatively more subseasonal variability.

Multiplicative noise has two opposing effects upon SST that almost exactly cancel, leading to little change between the two curves in Fig. 7b. First, the noise-induced drift weakens the effective damping of SST and thus acts to increase SST variance. If only this effect is removed, SST variance is reduced by about 10% (not shown), consistent with the results in SNA. However, removing multiplicative noise also reduces AIRT variance, so consequently the forcing of SST by the atmosphere is reduced. Note that the red-noise structure of the SST spectrum is not visibly altered by the multiplicative noise.

## 4.6 Other Ocean Weather Stations

We repeated the analysis performed upon OWS P for two other stations with shorter data records: OWS N in the North Pacific and OWS K in the North Atlantic (see Table 1 for the exact locations). These stations are both in midlatitudes and far away from strong currents. Therefore, we expect to see rather similar characteristics at OWS P, N, and K.

The observed joint PDFs of AIRT and SST at OWS K and N are shown on the left



hand sides of Figs. 8a and b. Indeed, the joint PDF at OWS K (Fig. 8a, left hand side) is quite similar to the PDF observed at OWS P (Fig. 6b), albeit with the SSTs slightly more kurtosed and skewed. The joint PDF at OWS N (Fig. 8b, left hand side) also shows a similar structure, but the deviation from Gaussianity is notably rotated so that the non-Gaussianity is much stronger in the direction of the SST-axis (consistent with the marginal SST PDFs shown in SNA).

The modeled joint PDFs of AIRT and SST at OWS K and N are shown on the right hand sides of Figs. 8a and b. It can be seen that the simple stochastic model is able to reproduce the joint PDF at OWS K relatively well (Fig. 8a), including the slight rotation of the PDF relative to OWS P. In fact, all the parameters at OWS K are broadly similar to the parameters determined at OWS P. At OWS N the model reproduces the basic structure, but captures only some of the rotation of the observed PDF (Fig. 8b). In the model, such clockwise rotation of the PDF anomaly occurs because of an increase in the estimated  $\gamma_a/\gamma_o$  ( $\approx 0.2$  at OWS N), which results in relatively more multiplicative noise forcing of SST [e.g., in (16)]. Although the failure of the model to completely reproduce the OWS N PDF might be due to the difficulty of estimating parameters from the shorter data record, it seems more likely a result of a deficiency of the model itself. Notably, the model combines latent and sensible heat fluxes together using the Bowen ratio formulation and then parameterizes the heat flux solely as a function of the air-sea temperature difference. Strictly speaking, however, the latent heat flux is a function of the dewpoint as well. Since OWS N is further south than OWS P and K, the latent heat flux there is relatively more important, and may need to be treated separately rather than combined with sensible heat flux as in our model.

Also, recall that the skew in this model is due to the mean stability parameter  $\Pi$ , which determines the amount of additive noise correlated with the multiplicative noise in  $\mathbf{B}_M(\mathbf{T}')$ . For the latent heat flux, the relevant parameter is instead the difference between the mean SST and mean dewpoint temperature. This difference is twice as large at OWS N as at OWS P, so the fraction of correlated additive noise might be underestimated by a model that focuses on the mean AIRT-SST difference alone.

The key point is that the multiplicative noise physics and the related non-Gaussianity at OWS P are not unique but also appear relevant at other midlatitude stations. This adds confidence to the ability of the model to capture the basic mechanism through which rapid wind variability impacts air-sea thermal coupling.

## 5 Summary and conclusions

In this paper we investigated how air-sea thermal coupling is impacted by the presence of rapidly varying surface winds, an effect that has generally been ignored in past studies. By representing the rapidly-varying winds as white-noise within a bulk formulation of surface turbulent heat fluxes, an equation for the coupled AIRT-SST system is derived that notably includes a stochastic process depending upon the atmospheric and oceanic thermal anomalies themselves. This state-dependent, or multiplicative, noise alters the predictable dynamics of atmosphere-ocean coupling because it induces an additional heat flux term, the noise-induced drift. In our model the noise-induced drift acts to effectively weaken coupling but also to effectively weaken dissipation. For example, at OWS P the noise-induced drift results in

about a one-third decrease of the effective coupling parameters, and about a 10-20% decrease of the effective damping parameters. In general, the effect of reduced thermal damping is weakened by noise in the coupling coefficients, one possible reason why the empirically determined effective coupling parameters for all three Ocean Weather Stations are much weaker than those used by BB98. Thus, while multiplicative noise acts to *increase* the persistence of SST and AIRT anomalies by reducing the damping of each, it simultaneously acts to *decrease* the persistence by weakening the air-sea coupling.

Our analysis has focused on how rapid wind variability results in multiplicative noise in the surface heat flux. Our model is quite simple, and in particular does not include any possible dynamical response of the atmosphere to SST anomalies, most notably the response of the surface wind variability itself. As SNA had previously found, explicit consideration of the latent flux dependence upon dewpoint, and bulk coefficients dependence upon stability, might also refine the model. Given these limitations, it is a striking result that the inclusion of this multiplicative noise term allows our model to reproduce not only the autocorrelation functions of SST and AIRT anomalies, but also the main non-Gaussian features of the observed joint PDFs. Moreover, our model predicts that the multiplicative noise will be linearly dependent upon the surface air-sea temperature difference, a result that is confirmed by observations. This agreement with observations suggests that our model captures the basic impact of rapid wind variability upon air-sea thermal coupling.

An important consequence of the theory outlined here is that air-sea coupling includes both deterministic and stochastic components. That is, rapidly varying boundary-layer winds and related heat fluxes are unpredictable on the longer dynamical time-scales of the

ocean yet are a crucial component of atmosphere-ocean coupling, and may not be just averaged away. Note that by modeling winds as "rapid" white-noise, our analysis can not determine the relative importance of synoptic variability of the wind field versus mesoscale variability and wind gustiness. Still, there is clearly a significant scale interaction between the fast wind induced heat flux variability and the slower AIRT and SST variability. This scale interaction has implications for coupled models, as rapid variability of boundary layer winds and related surface fluxes must be accurately simulated to correctly model not only AIRT and SST variability but also the mean climate, which is also affected by the noise-induced drift (e.g., Sardeshmukh et al. 2001). That is, a coupled model with incorrect atmospheric variability (i.e., noise) might incorrectly estimate the strength of atmosphere-ocean coupling. For example, many atmospheric models (but not all) significantly underestimate wind variability in the marine boundary layer (e.g., Gille 2005; Monahan 2006a,b). It is unclear how a bias in wind variability (and related heat fluxes) contributes to biases found in many coupled models. However, we have a powerful theory/tool at hand to systematically study this problem. The results could guide the development of improved coupling schemes and the implementation of stochastic parameterizations in climate models.

*Acknowledgments.* This work was supported by a grant from NOAA CLIVAR/Pacific and NSF grant 0552047.

# Appendix A

## Stochastic dynamics in a nutshell

This appendix reviews a few basic ideas of stochastic dynamics used in this paper. More comprehensive treatments may be found in many textbooks (e.g., Gardiner 2004; Horsthemke and L  f  ver 1984; Paul and Baschnagel 1999).

Consider the dynamics of an  $n$ -dimensional system whose state vector  $\mathbf{x}$  is governed by the stochastic differential equation (SDE)

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\boldsymbol{\eta} \quad (\text{A.1})$$

where the vector  $\mathbf{A}$  represents all slow processes and  $\mathbf{B}(\mathbf{x})\boldsymbol{\eta}$ , with the matrix  $\mathbf{B}$  and the noise vector  $\boldsymbol{\eta}$ , represents the stochastic approximation to the fast nonlinear processes. The stochastic components  $\eta_i$  are assumed to be independent Gaussian white noise processes:

$$\langle \eta_i(t) \rangle = 0, \quad \langle \eta_i(t) \eta_j(t') \rangle = \delta_{ij} \delta(t - t') \quad (\text{A.2})$$

where  $\langle \dots \rangle$  denotes the averaging operator. The corresponding Fokker-Planck equation,

$$\begin{aligned} \frac{\partial p(\mathbf{x}, t)}{\partial t} = & - \sum_i \frac{\partial}{\partial x_i} \left[ A_i + \alpha \sum_{j,k} \left( \frac{\partial}{\partial x_j} B_{ik} \right) B_{jk} \right] p(\mathbf{x}, t) \\ & + \frac{1}{2} \sum_{i,j} \frac{\partial^2}{\partial x_i \partial x_j} (\mathbf{B} \mathbf{B}^T)_{ij} p(\mathbf{x}, t), \end{aligned} \quad (\text{A.3})$$

describes the conservation of the probability density  $p(\mathbf{x}, t)$  of the system described by the SDE (A.1). Two different values of  $\alpha$  yield two physically important stochastic calculi: the It   ( $\alpha = 0$ ) and the Stratonovich calculus ( $\alpha = 1/2$ ). On the right hand side, the first term within square brackets describes the dynamics of the deterministic system and is called the

deterministic drift. The second term within square brackets, which does not occur in Itô systems ( $\alpha = 0$ ), is called the noise-induced drift. The remaining term is associated with the diffusion of the probability density by noise.

For a detailed discussion of stochastic integration and the differences between Itô and Stratonovich SDEs see for example Horsthemke and L  f  ver (1984), Gardiner (2004), or Penland (1996). The key point here is that the Stratonovich calculus is relevant for continuous physical systems, such as the atmosphere, in which rapidly fluctuating quantities with small but finite correlation times are approximated as white noise. Thus, simplified stochastic models constructed from atmospheric dynamical equations may assume Stratonovich calculus. However, if instead a stochastic model is indirectly estimated from observed discrete data, then the inferred drift will be the *sum* of the deterministic and the noise-induced drifts. In this case using the It   framework may be preferable, where now  $\mathbf{A}(\mathbf{x})$  represents not just the deterministic drift but rather this sum, or the “effective drift”.

Equations for moments of  $\mathbf{x}$  can be obtained by multiplying the Fokker-Planck equation (A.3) by powers of  $\mathbf{x}$  and integrating over all  $\mathbf{x}$ . In particular, second moments of  $\mathbf{x}$  are given by

$$\frac{d\langle \mathbf{x} \mathbf{x}^T \rangle}{dt} = \langle \mathbf{A}(\mathbf{x}) \mathbf{x}^T \rangle + \langle \mathbf{x} \mathbf{A}^T(\mathbf{x}) \rangle + \langle \mathbf{B}(\mathbf{x}) \mathbf{B}^T(\mathbf{x}) \rangle. \quad (\text{A.4})$$

This equation is known as the fluctuation-dissipation relation (FDR) of the system (see e.g., Penland and Matrosova 1994).

In principle, the deterministic and stochastic parts of (A.3) can be determined from data by using their statistical definitions (Siegert et al. 1998; Friedrich et al. 2000; Gradi  sek et al.

2000; Sura and Barsugli 2002; Sura 2003; Sura and Gille 2003; Sura et al. 2006):

$$\mathbf{A}(\mathbf{x}) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \langle \mathbf{X}(t + \Delta t) - \mathbf{x} \rangle |_{\mathbf{x}(t)=\mathbf{x}} \quad (\text{A.5})$$

$$\mathbf{B}(\mathbf{x})\mathbf{B}^T(\mathbf{x}) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \langle (\mathbf{X}(t + \Delta t) - \mathbf{x})(\mathbf{X}(t + \Delta t) - \mathbf{x})^T \rangle |_{\mathbf{x}(t)=\mathbf{x}} \quad (\text{A.6})$$

where  $\mathbf{X}(t + \Delta t)$  is a solution (a single stochastic realization) of the SDE (A.1) with the initial condition  $\mathbf{X}(t) = \mathbf{x}$  at time  $t$ . The data define a state space representing every observed value of  $\mathbf{x}$ . The effective drift and stochastic diffusion are estimated by replacing the theoretical limit  $\Delta t \rightarrow 0$  with a finite-difference approximation. In practice, estimating  $\mathbf{B}(\mathbf{x})\mathbf{B}^T(\mathbf{x})$  from discretely sampled data is prone to error, because Taylor expansions of stochastic terms are proportional to  $\sqrt{\Delta t}$  and not proportional to  $\Delta t$  as are the deterministic terms (e.g., Sura and Barsugli 2002; Sura 2003). Note that  $\mathbf{B}(\mathbf{x})\mathbf{B}^T(\mathbf{x})$  rather than  $\mathbf{B}(\mathbf{x})$  is estimated from data. In general it is impossible to find a unique expression for  $\mathbf{B}(\mathbf{x})$  in the multivariate case (e.g., Monahan 2004).

When  $\mathbf{A}$  and  $\mathbf{B}$  are known, analytical solutions of the Fokker-Planck equation (A.3) for  $p(\mathbf{x}, t)$  can only be found in limited cases (appendix B presents one such case). For more general cases, numerical methods must be used. To interpret the results of the Fokker-Planck equation, numerical integrations of the SDE (A.1) can also be performed.



# References

- Alexander, M. A., Blade, I., Newman, M., Lanzante, J. R., Lau, N.-C., and Scott, J. D., 2002: The atmospheric bridge: The influence of ENSO teleconnections on air-sea interaction over the global ocean. *J. Climate*, **14**, 2205–2231.
- Alexander, M. A., and Penland, C., 1996: Variability in a mixed layer ocean model driven by stochastic atmospheric forcing. *J. Climate*, **9**, 2424–2442.
- Alexander, M. A., and Scott, J. D., 1997: Surface flux variability over the North Pacific and North Atlantic oceans. *J. Climate*, **10**, 2963–2978.
- Azzalini, A., 2005: *R package 'sn': The skew-normal and skew-t distribution (version 0.33)*. Universit di Padova, Italia.
- Azzalini, A., and Capitanio, A., 2003: Distributions generated by perturbations of symmetry with emphasis on a multivariate skew-*t* distribution. *J. R. Statist. Soc. B*, **65**, 367–389.
- Barsugli, J. J., and Battisti, D. S., 1998: The basic effects of atmosphere-ocean thermal coupling on midlatitude variability. *J. Atmos. Sci.*, **55**, 477–493.
- Frankignoul, C., and Hasselmann, K., 1977: Stochastic climate models. Part II. Application to sea-surface temperature anomalies and thermocline variability. *Tellus*, **29**, 289–305.
- Frankignoul, C., and Kestenare, E., 2002: The surface heat flux feedback. Part I: Estimates from observations in the Atlantic and the North Pacific. *Climate Dynamics*, **19**, 633–647.
- Friedrich, R., Siegert, S., Peinke, J., Lück, S., Siefert, M., Lindemann, M., Raethjen, J.,

- Deusch, G., and Pfister, G., 2000: Extracting model equations from experimental data. *Phys. Lett. A*, **271**, 217–222.
- Gardiner, C. W., 2004: *Handbook of Stochastic Methods for Physics, Chemistry and the Natural Science, Third Edition*. Springer-Verlag, 415 pp.
- Gille, S. T., 2005: Statistical characterization of zonal and meridional ocean wind stress. *J. Atmos. Ocean. Tech.*, **22**, 1353–1372.
- Gradišek, J., Siegert, S., Friedrich, R., and Grabec, I., 2000: Analysis of time series from stochastic processes. *Phys. Rev. E*, **62**, 3146–3155.
- Hall, A., and Manabe, S., 1997: Can local linear stochastic theory explain sea surface temperature and salinity variability? *Climate Dyn.*, **13**, 167–180.
- Hasselmann, K., 1976: Stochastic climate models. Part I. Theory. *Tellus*, **28**, 473–484.
- Horsthemke, W., and Léfever, R., 1984: *Noise-Induced Transitions: Theory and Applications in Physics, Chemistry, and Biology*. Springer-Verlag, 318 pp.
- Jones, M. C., and Faddy, M. J., 2003: A skew extension of the  $t$ -distribution, with applications. *J. R. Statist. Soc. B*, **65**, 159–174.
- Kloeden, P., and Platen, E., 1992: *Numerical Solution of Stochastic Differential Equations*. Springer-Verlag, 632 pp.
- Kraus, E. B., and Businger, J. A., 1994: *Atmosphere-Ocean Interaction*. Oxford University Press, 362 pp.

- Large, W. G., and Pond, S., 1982: Sensible and latent heat flux measurements over the ocean. *J. Phys. Oceanogr.*, **12**, 464–482.
- Monahan, A. H., 2004: A simple model for the skewness of global sea-surface winds. *J. Atmos. Sci.*, **61**, 2037–2049.
- Monahan, A. H., 2006a: The probability distributions of sea surface wind speeds Part I: Theory and SSM/I observations. *J. Climate*, **19**, 497–520.
- Monahan, A. H., 2006b: The probability distributions of sea surface wind speeds Part II: Dataset intercomparison and seasonal variability. *J. Climate*, **19**, 521–534.
- Mosedale, T. J., Stephenson, D. B., and Collins, M., 2005: Atlantic atmosphere-ocean interaction: A stochastic climate model-based diagnosis. *J. Climate*, **18**, 1086–1095.
- Park, S., Deser, C., and Alexander, M. A., 2005: Estimation of the surface heat flux response to sea surface temperature anomalies over the global ocean. *J. Climate*, **18**, 4582–4599.
- Paul, W., and Baschnagel, J., 1999: *Stochastic Processes: From Physics to Finance*. Springer-Verlag, 231 pp.
- Peinke, J., Böttcher, F., and Barth, S., 2004: Anomalous statistics in turbulence, financial markets and other complex systems. *Ann. Phys.*, **13**, 450–460.
- Penland, C., 1996: A stochastic model of Indo Pacific sea surface temperature anomalies. *Phys. D*, **98**, 543–558.
- Penland, C., and Matrosova, L., 1994: A balance condition for stochastic numerical models with application to El Niño - the Southern Oscillation. *J. Climate*, **7**, 1352–1372.

- R Development Core Team, 2004: *R: A language and environment for statistical computing*.
- R Foundation for Statistical Computing, Vienna, Austria, ISBN 3-900051-07-0.
- Ronca, R. E., and Battisti, D. S., 1997: Anomalous sea surface temperatures and local air-sea energy exchange on intraannual timescale in the northeastern subtropical Pacific. *J. Climate*, **10**, 102–117.
- Sardeshmukh, P., Penland, C., and Newman, M., 2001: Rossby waves in a fluctuating medium. P. Imkeller, and J.-S. von Storch, Eds., *Progress in Probability, Vol. 49: Stochastic Climate Models*, Birkhäuser Verlag, Basel.
- Sardeshmukh, P. D., Penland, C., and Newman, M., 2003: Drift induced by multiplicative red noise with application to climate. *Europhysics Letters*, **63**, 498–504.
- Siebert, S., Friedrich, R., and Peinke, J., 1998: Analysis of data sets of stochastic systems. *Phys. Lett. A*, **243**, 275–280.
- Sura, P., 2003: Stochastic analysis of Southern and Pacific Ocean sea surface winds. *J. Atmos. Sci.*, **60**, 654–666.
- Sura, P., and Barsugli, J. J., 2002: A note on estimating drift and diffusion parameters from timeseries. *Phys. Lett. A*, **305**, 304–311.
- Sura, P., and Gille, S. T., 2003: Interpreting wind-driven Southern Ocean variability in a stochastic framework. *J. Mar. Res.*, **61**, 313–334.
- Sura, P., Newman, M., and Alexander, M. A., 2006: Daily to decadal sea surface temperature

variability driven by state-dependent stochastic heat fluxes. *J. Phys. Oceanogr.*, **36**, 1940–1958.

Sura, P., Newman, M., Penland, C., and Sardeshmukh, P. D., 2005: Multiplicative noise and non-Gaussianity: A paradigm for atmospheric regimes? *J. Atmos. Sci.*, **62**, 1391–1409.

Table 1: The Ocean Weather Stations (OWSs) used for this study.

OWS	Location	Period
P	50°N, 145° W	1949–1981
N	30°N, 140° W	1946–1974
K	45°N, 16° W	1949–1975

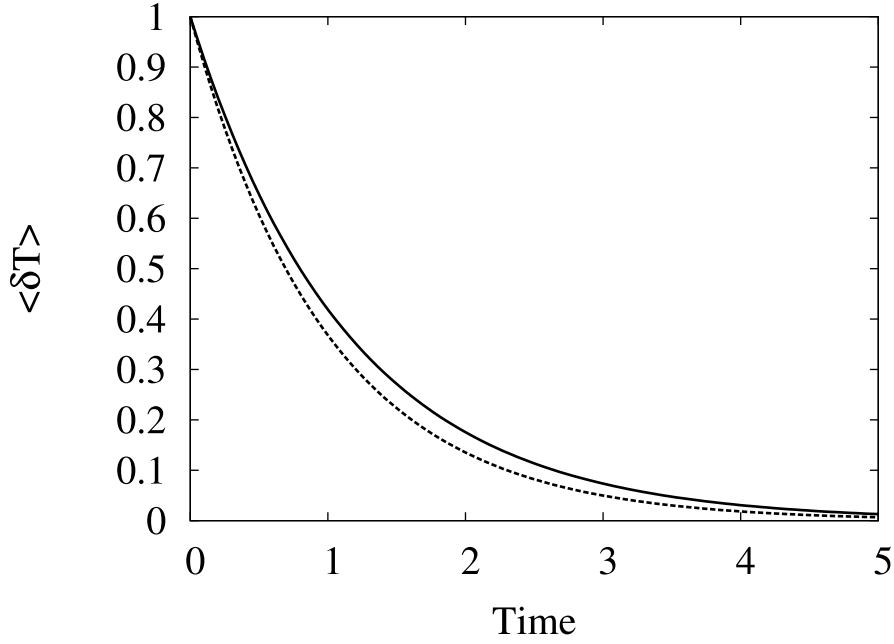
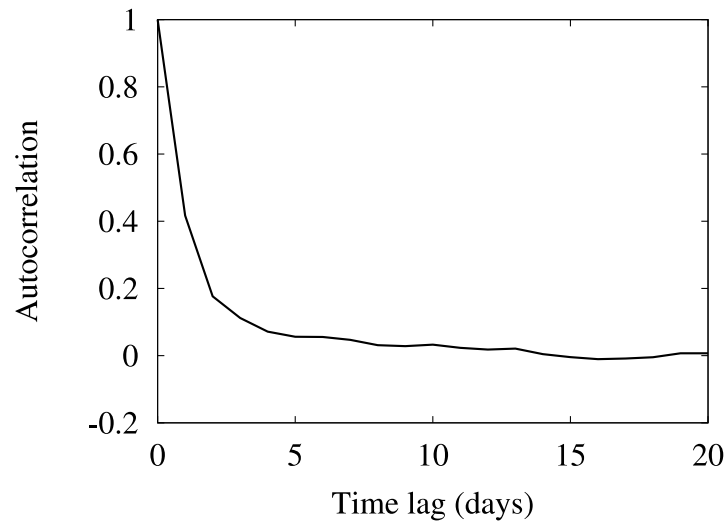


Figure 1: Solution of Eq. 6 (response with anomalous wind speeds; solid line) and Eq. 7 (response without anomalous wind speeds; dashed line) for  $\overline{|\mathbf{U}|} \equiv 1$ ,  $\sigma_{|\mathbf{U}|'} \equiv 0.5$ , and  $\delta T_o \equiv 1$ . It can be seen that with anomalous wind speeds  $\langle \delta T \rangle$  decays slightly slower than in the case with the mean wind only. That is, in this particular case (linear system with linear multiplicative noise) the stochastic forcing effectively reduces the damping of the system.

a)



b)

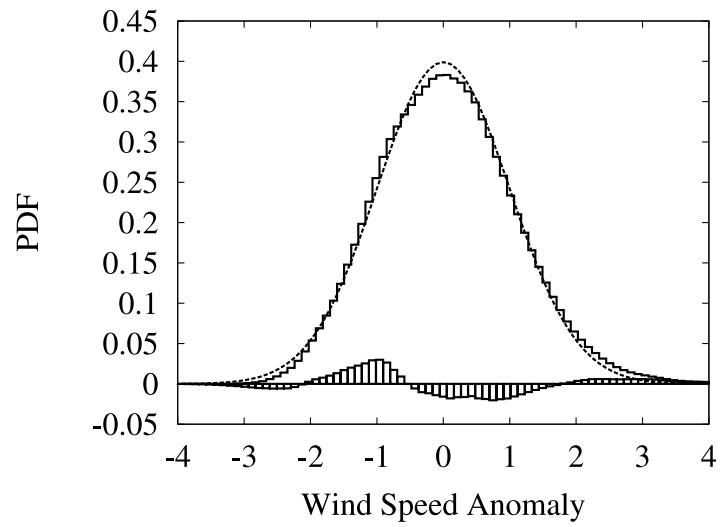
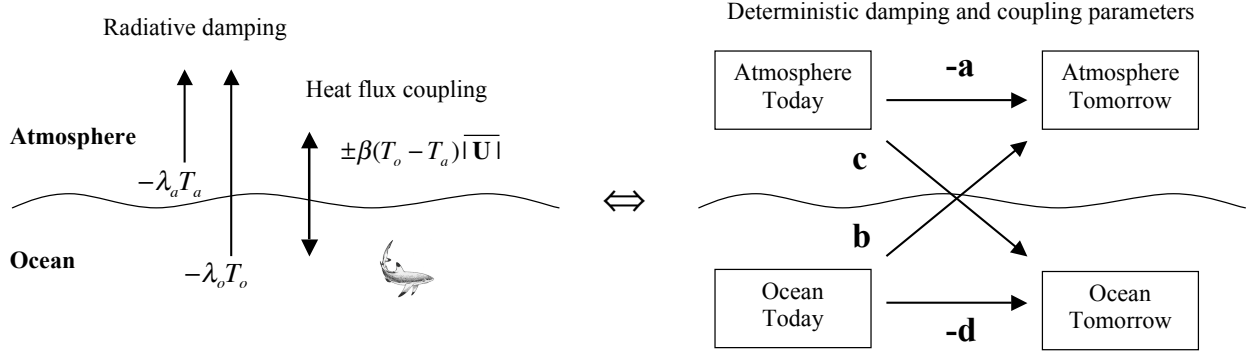


Figure 2: (a) Autocorrelation function and (b) PDF of wind speed anomalies (m/s) at OWS P.



### Air-Sea Interaction with Mean Wind only



### Air-Sea Interaction with Mean and Fluctuating Winds

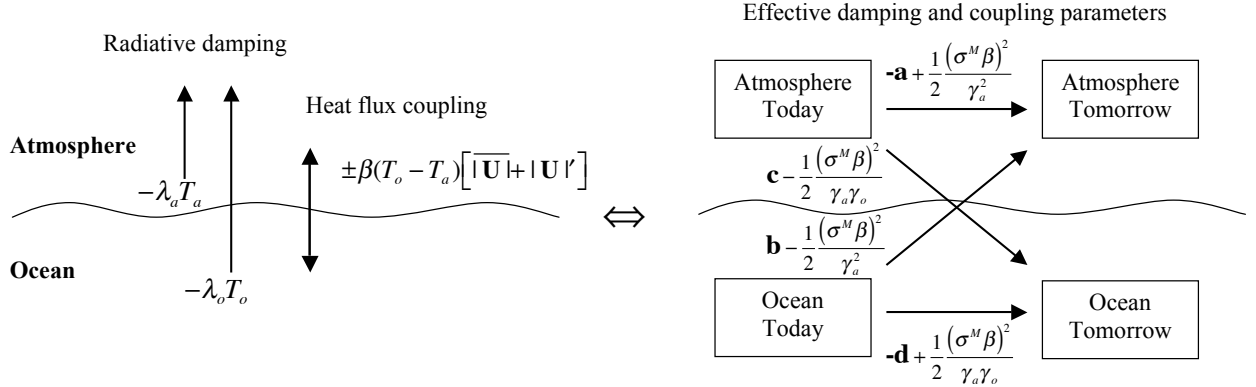
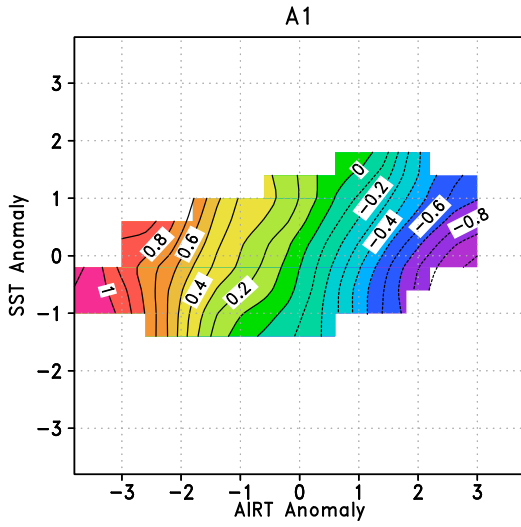
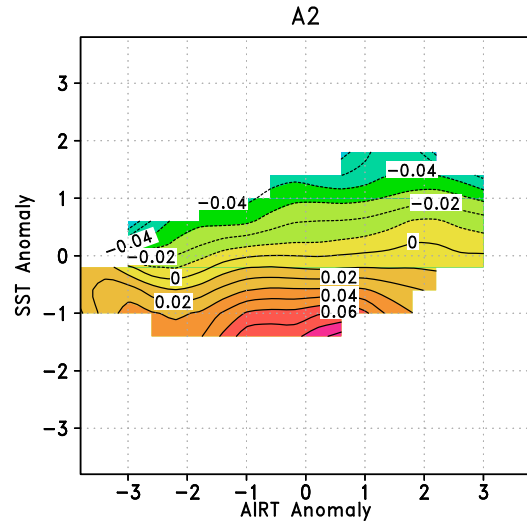


Figure 3: A schematic drawing to illustrate the physics/mathematics of our coupled model. We focus on the distinction between *air-sea interaction with mean wind only* (upper panel) and *air-sea interaction with mean and fluctuating winds* (lower panel). The physical set-ups are shown on the left hand sides of each panel, while the equivalent mathematical relations are depicted on the panel's right hand sides. In the case with only the mean wind the evolution of atmosphere-ocean temperatures, given by the deterministic drift matrix with parameters  $-a, b, c, -d$ , is visualized by the corresponding arrows:  $-a$  damps the atmosphere,  $b$  is the oceanic forcing of the atmosphere,  $c$  is the atmospheric forcing of the ocean, and  $-d$  damps the ocean. Most importantly, note that in case were the heat flux is due to the mean and fluctuating winds the evolution of atmosphere-ocean temperatures, given by the effective drift matrix with parameters  $-a + \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a^2}, b - \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a^2}, c - \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a \gamma_o}, -d + \frac{1}{2} \frac{(\sigma^M \beta)^2}{\gamma_a \gamma_o}$ , is altered by the noise-induced drift. The noise effectively decreases the absolute values of the damping and coupling parameters, altering the dynamics of air-sea interaction. See section 3 for a detailed discussion.

a)



b)



c)

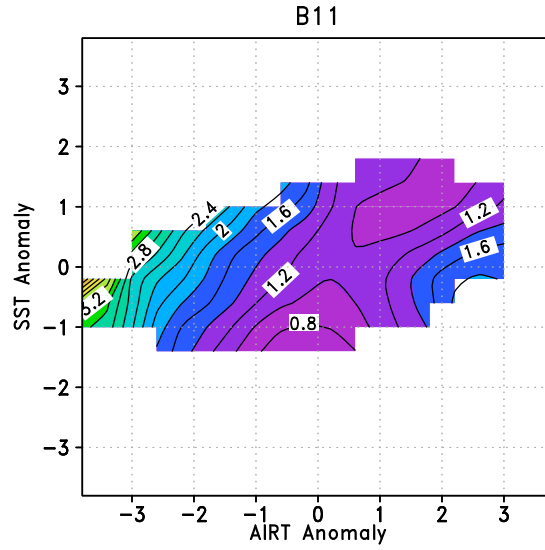
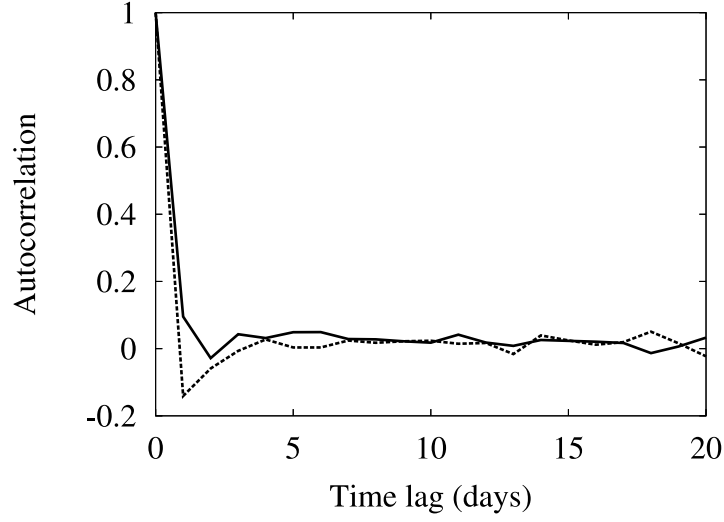


Figure 4: a-b) The effective drift  $\mathbf{A}(\mathbf{x})$  estimated from AIRT and SST anomalies at OWS P. A1 denotes the  $T'_a$  component (atmosphere) and A2 denotes the  $T'_o$  component (ocean) of the two-dimensional system. c)  $(\mathbf{B}\mathbf{B}^T)_{11}$  element of the noise matrix  $\mathbf{B}\mathbf{B}^T$ . See section 4.2 for details.

a)



b)

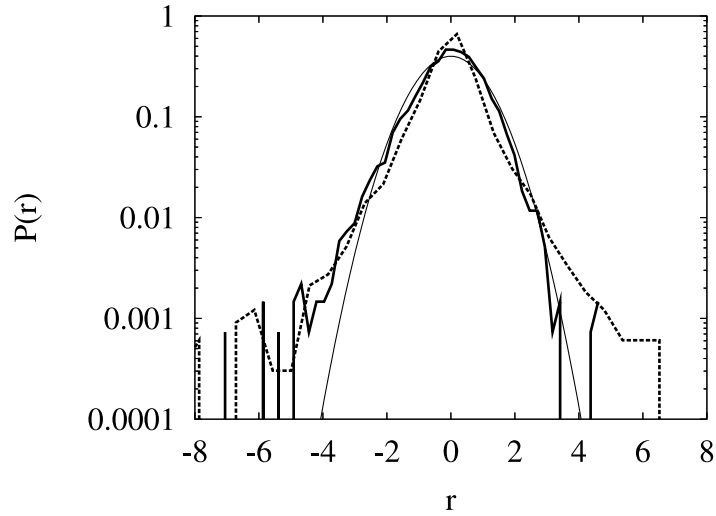
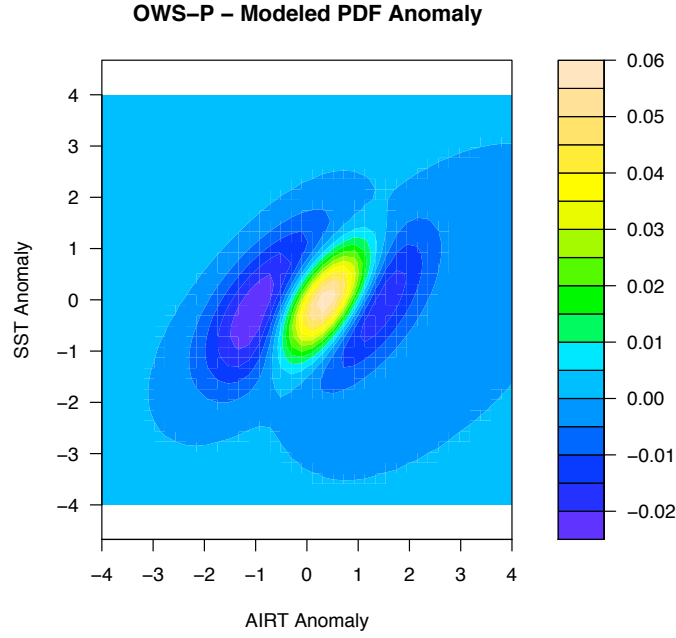


Figure 5: Autocorrelation functions (a) and PDFs (b) of the residual  $\mathbf{r} \equiv \mathbf{T}'_{obs}(t+\Delta t) - \mathbf{T}'(t+\Delta t)$  for air temperature anomalies (thick solid lines) and sea surface temperature anomalies (dashed lines). Note that for both components the autocorrelation is close to zero after one time step of 1 day, and that the residuals are highly non-Gaussian (the thin solid line in (b) shows the related Gaussian distribution). See section 4.3 for details.

a)



b)

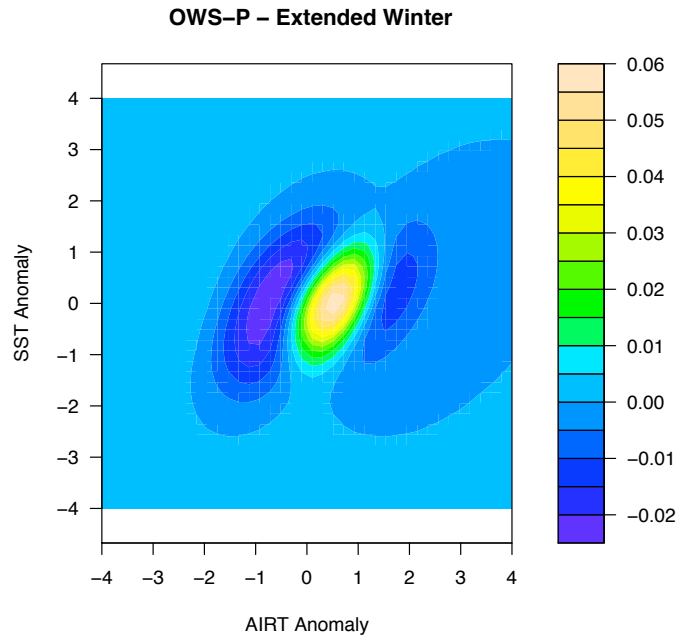
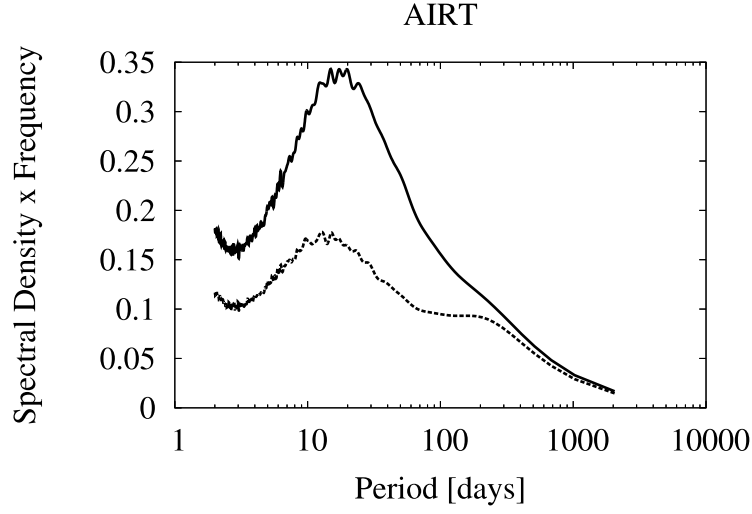


Figure 6: a) Modeled joint PDF anomalies (deviations from Gaussianity) of AIRT and SST anomalies with parameters for OWS P. b) Observed joint PDF anomalies (deviations from Gaussianity) of AIRT and SST anomalies at OWS P. Here AIRT and SST anomalies are normalized to have zero mean and unit standard deviation.

a)



b)

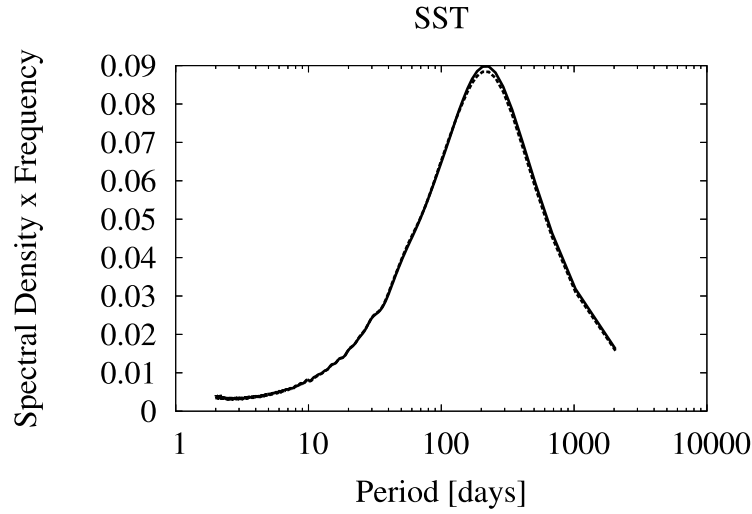
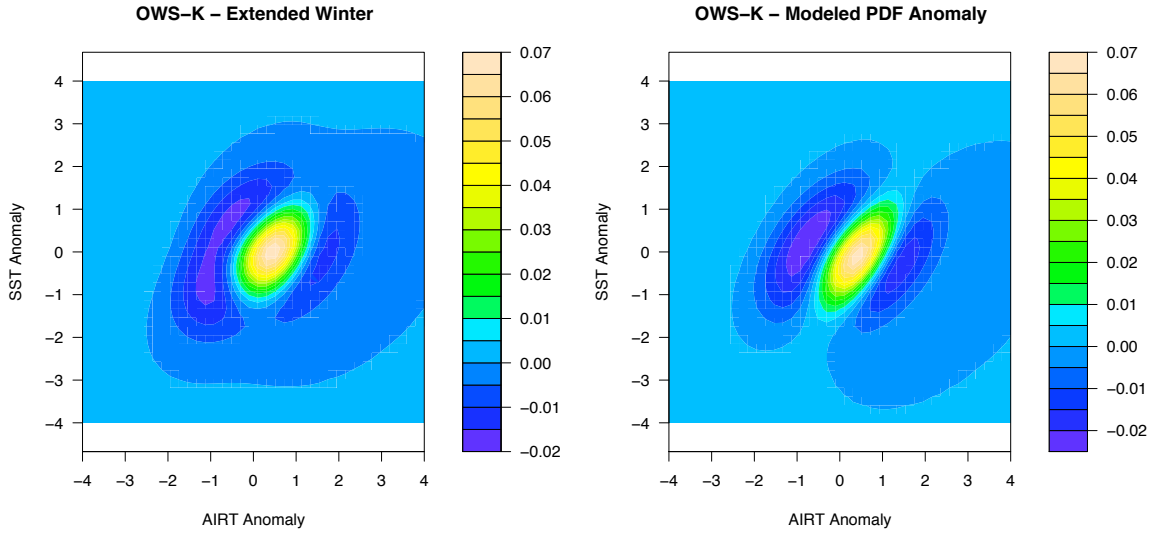


Figure 7: Spectra of modeled anomalous (a) AIRT and (b) SST variability without and with multiplicative noise for the first set of experiments (multiplicative noise turned off without accounting for lost variance). The spectra with pure additive noise are indicated by the dashed lines, and the spectra with multiplicative noise included are indicated by the solid lines.

b) OWS K ( $45^\circ$  N,  $16^\circ$  W)



a) OWS N ( $30^\circ$  N,  $140^\circ$  W)

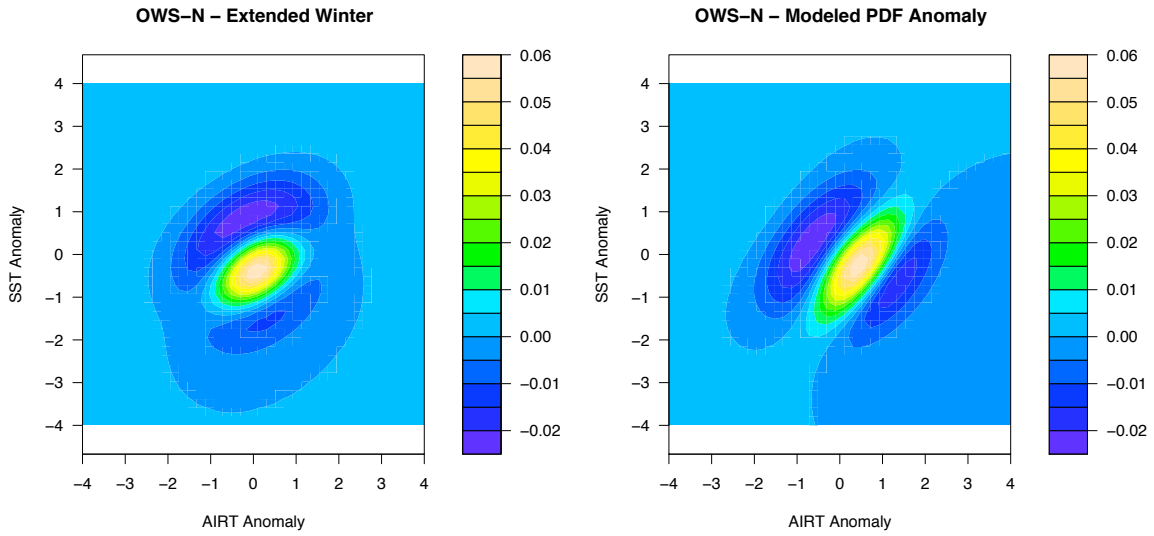


Figure 8: Observed (left) and modeled (right) joint PDF anomalies (deviations from Gaussianity) of AIRT and SST anomalies at a) OWS K, and b) OWS N. Note that while the contour interval is the same in this plot and in Fig. 6, color shading is different for each station. Here AIRT and SST anomalies are normalized to have zero mean and unit standard deviation.